

DEVELOPABLE SQUARE LATTICES

A constrained geometric system for coupling curvature, metric compatibility, and constructible sheet organization

Baha Odaibat | Geometric Investigation Artifact | 2026

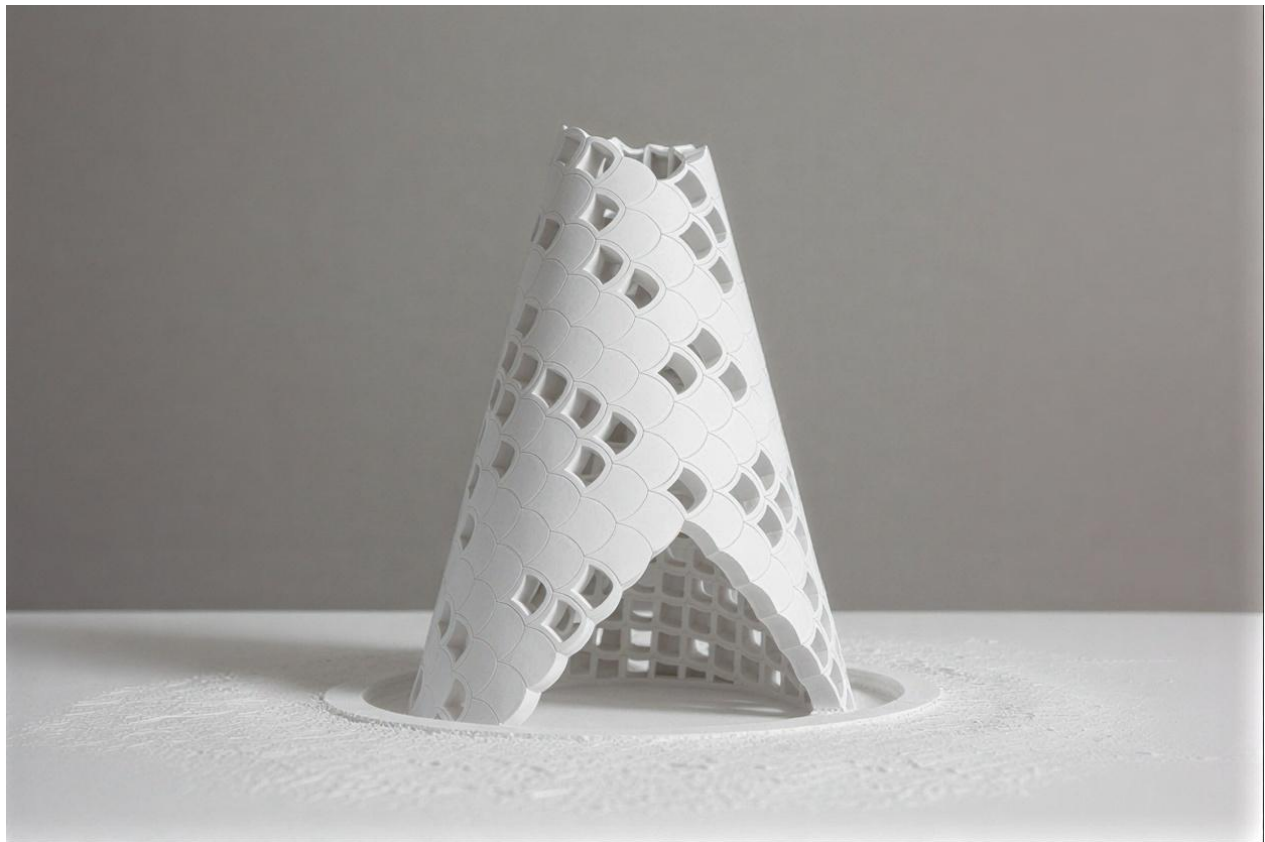


Fig. 01 - Pavilion manifestation of the developable square lattice system.

Abstract

This investigation defines a constrained geometric system in which a uniform square metric remains compatible with a family of curved developable surfaces under discrete closure conditions.

The investigation begins through physical band experiments that reveal discrete closure behavior governed by an integer parameter N . These states transition between deficit conditions, a flat equilibrium state, and surplus oscillatory geometries.

The result is a constrained geometric domain in which curvature, metric compatibility, developability, and constructibility are generated through the same admissibility framework.

1. Research Positioning

Constraint Before Form

The investigation defines a constraint-based geometric system rather than a freeform surface exploration. Geometry is admitted or rejected according to whether it satisfies a metric condition.

CONVENTIONAL WORKFLOW	THIS INVESTIGATION
Freeform geometry first	Metric admissibility first
Rationalization after form generation	Rationality generates form
Fabrication as correction	Fabrication as geometric condition
Surface freedom prioritized	Constraint system prioritized

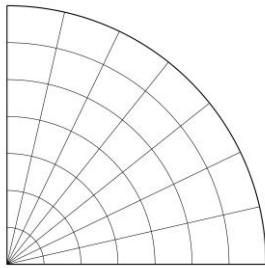
The square metric is not applied as a secondary subdivision system. It acts as a governing geometric condition determining which curved states are admissible within the constructible family.

2. Research Contribution

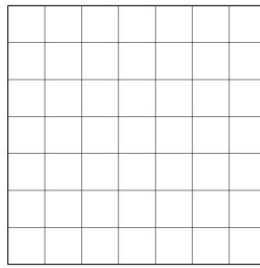
CONTRIBUTION	GEOMETRIC CONSEQUENCE	ARCHITECTURAL IMPLICATION
Metric preservation	Uniform square metric preserved under developable bending	Constructibility embedded at system level
Quantized states	Discrete deficit/flat/surplus regimes	Legible geometric family
Mode organization	Discrete branch differentiation	Multiple admissible spatial states
Developable curvature	Developable metric surface family	Transferable tectonic systems
Architectural	Metric enclosure organization	Spatial and tectonic instantiation

3. Problem Definition

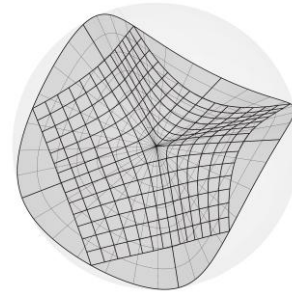
Square Metric Versus Radial Geometry



Radial organization



Square metric



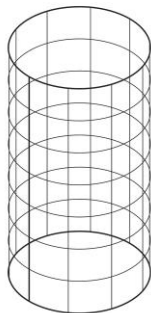
Radial/Metric Overlay

SURFACE / SYSTEM	GEOMETRIC CONDITION	ROLE IN RESEARCH
Cylinder	Developable; linear development	Geometrically trivial case
Cone	Developable; radial organization	Critical metric conflict case
Dome	Double curvature; non-developable	Metric preservation impossible
This research	Square metric + developable curvature	Defines admissible curved states

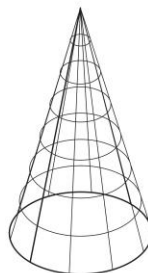
The conflict is not resolved by following the cone’s radial subdivision. It is resolved by introducing a square metric that cuts across the radial organization. The lattice therefore acts as a metric correction to the cone’s native parameterization: curvature is allowed through bending, while subdivision density remains governed by the square grid rather than by radial convergence.

The cone therefore becomes the minimum geometric condition in which curvature, developability, and metric incompatibility coexist simultaneously within the same constructible surface system.

Comparative geometric evidence: developable states versus non-developable curvature.



Cylinder as rolled linear development.



Cone as radial rolled development.



Dome as non-developable condition.

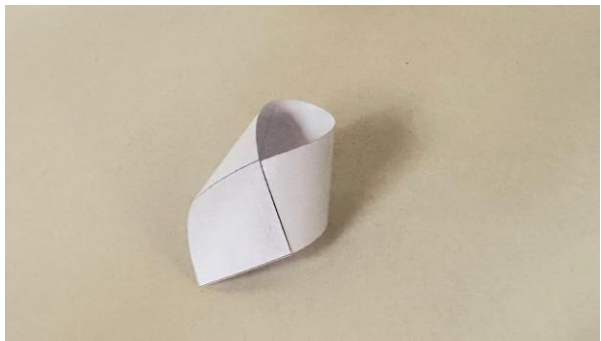
4. Physical Discovery

Orthogonal Closure and the Emergence of N

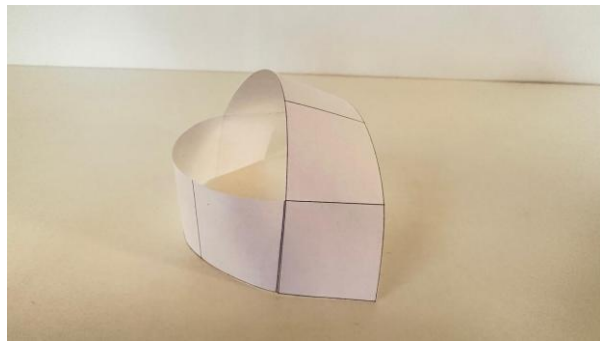
The physical paper-band models demonstrate that discrete geometric states emerge directly from orthogonal closure under metric constraint. The admissible configurations appear physically before computational reconstruction, revealing that the system is governed by closure behavior rather than by arbitrary formal variation.

N does not function as a resolution parameter or subdivision count. It defines the discrete global closure condition of the system itself. Changes in N therefore produce different admissible geometric states rather than finer approximations of the same geometry.

Physical evidence strip: discrete closure behavior observed before computation.



N1/ Single band / Deficit regime



N2/ Double band / Deficit regime



N3/ Triple band / Deficit regime



N5/ lobe formation / Surplus regime

5. Mathematical Lineage $X(r,s) = r u(s)$

Spherical Elastica and Developable Geometry

The system builds upon spherical elastica formulations associated with e-cone equilibrium states and developable ruled surface geometry. The present work does not attempt to derive these formulations analytically; rather, it investigates their geometric and constructive consequences under square-metric conditions.

The admissible geometries correspond to equilibrium states described within the e-cone literature, where curvature organization emerges through constrained spherical elastica behavior.

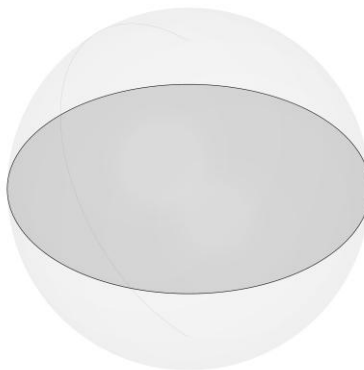
A closed spherical director curve defines the developable ruled surface. Each point on the surface is generated by scaling a direction $u(s)$ from the apex, so the surface remains ruled by construction. Curvature organization is encoded in the spherical director curve rather than imposed through arbitrary surface deformation.

MATHEMATICAL CONCEPT	ROLE IN SYSTEM
Closed spherical curve $u(s)$	Director curve for developable ruled surface
Surplus angle / e-cone behavior	Transition toward lobed states
Discrete solution branches	Mode-controlled curvature organization
Fixed metric condition	Square metric preserved under bending
Closure constraints	Quantized admissible states

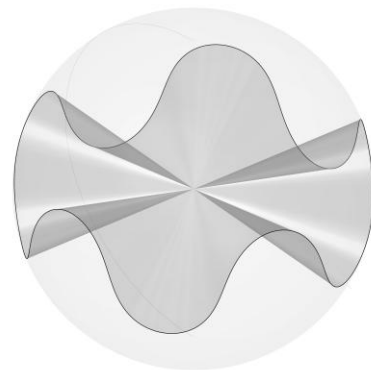
Deficit, planar, and surplus curvature states represented through spherical elastica configurations.



sub-equatorial



equatorial



super-equatorial

6. Discrete Geometric System

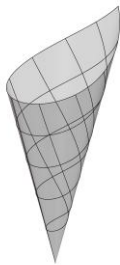
N, Mode, and the Elliptic Parameter

PARAMETER	FUNCTION	GEOMETRIC EFFECT	ARCHITECTURAL READING
N	Quantized closure condition	Deficit/flat/surplus transition	Position within family
mode	Equilibrium branch index	Discrete lobe configuration	Global spatial definition
m	Elliptic parameter	Closure consistency	Hidden mathematical tuning
samples	Curve reconstruction	Resolution precision	Documentation fidelity
surfaceSteps	Ruled surface resolution	Surface reconstruction	documentation

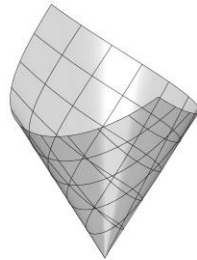
The system operates through a small parameter set while producing discrete but spatially differentiated geometric states.

6.5 Admissible Geometric Family

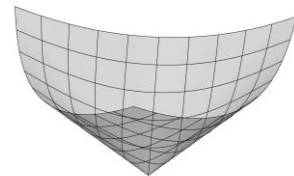
Discrete family progression generated through the developable square lattice system.



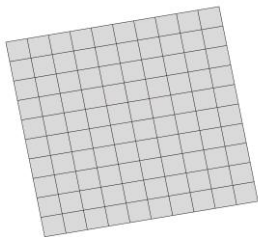
N1/Deficit state.



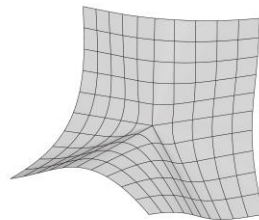
N2/Deficit state.



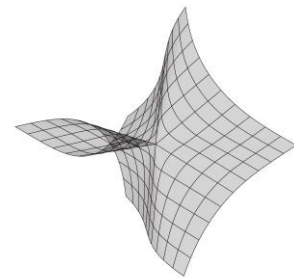
N3/ Deficit state.



N4/ Flat inversion state.

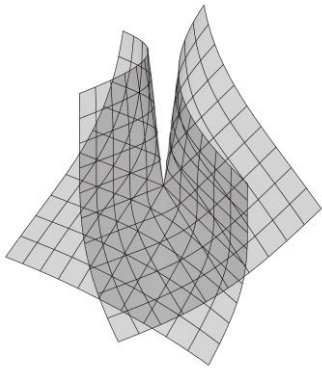


N5/ Surplus emergence.

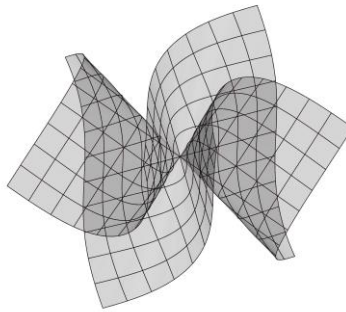


N6/ Advanced surplus family.

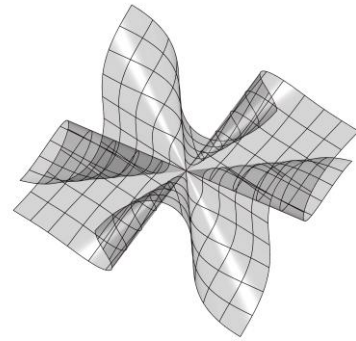
Mode variation within a fixed N-state: multiple admissible equilibrium branches under the same global closure condition.



N8/Mode2



N8/Mode3



N8/Mode5

7. Position Within the Research Landscape

Its distinction lies in treating metric compatibility as a generative geometric condition rather than as a post-rationalization problem.

“How can metric compatibility operate as a generative condition for admissible curved geometry?”

8. System Implications

8.1 Geometric Organization

The system defines admissible curvature states through a discrete N/mode structure. Variation occurs within a bounded geometric field rather than through unrestricted surface freedom. Each state belongs to the same metric family while producing different spatial organizations of curvature, enclosure, and lobe formation.

8.2 Fabrication Logic

The square lattice defines the transferable construction unit of the system. Surface behavior, panelization, rib structures, apertures, and sheet organization emerge as different material readings of the same metric subdivision rather than as independent geometric systems.

8.3 Architectural Consequences

The pigeon tower / pavilion demonstrates a direct architectural instantiation of the geometric system. The square lattice does not remain a surface diagram; it becomes aperture distribution, enclosure logic, light modulation, structural continuity, and inhabitable organization. These architectural effects emerge from the same metric subdivision rather than from secondary formal manipulation.

9. System Properties

9.1 Rationality as Generator

Rationality defines admissibility before form generation.

9.2 Small Grammar, Large Consequences

A minimal parameter set generates a discrete but expandable family of admissible geometric states.

9.3 Developable Curvature as Middle Ground

The system defines a constrained geometric domain in which curvature remains developable while metric organization remains controlled.

9.4 Transition at $N = 4$

$N = 4$ defines the equilibrium inversion state of the system: the critical transition between deficit and surplus curvature regimes. At this state, curvature organization reaches planar resolution before bifurcating into oscillatory surplus behavior.

9.5 Lattice as Tectonic Language

The lattice operates simultaneously as geometric subdivision, structural organization, aperture logic, and fabrication framework. Because the metric condition remains uniform across the surface, these systems remain continuously compatible without reverting to radial hierarchy or local geometric exceptions.

10. Research Claims

1. A uniform square metric can be preserved across a family of curved developable surfaces under quantized closure conditions.
2. N functions as a geometric state parameter rather than a resolution parameter.
3. Mode selects admissible folded branches controlling lobe organization.
4. Fabrication rationality operates as a governing geometric condition at the level of form generation itself.
5. Architectural consequence can emerge directly from the same metric logic, as demonstrated through the pigeon tower / pavilion case.



11. Limits and Open Questions

QUESTION	AFFECTED DOMAIN	FUTURE DIRECTION
Structural stability across N/mode	Structural behavior	Simulation and testing
Thickness and self-contact	Material constraints	Tolerance studies
Manufacturable panel systems	Fabrication	Edge and joint development
Environmental performance	Acoustics / lighting	Performance integration
Altitude-biased curvature distribution	Global geometric organization	Non-symmetric spherical elastica formulations
Localized control of lobe intensity at (t)	Local curvature behavior	Weighted boundary-condition systems
Expansion beyond apex systems	Geometric family	Annular and multi-apex systems

12. Conclusion

This investigation defines a constrained geometric domain in which curvature, metric compatibility, developability, and constructibility emerge from the same admissibility structure.

The work does not propose unrestricted surface freedom. It defines a discrete family of admissible geometric states governed by closure conditions, metric compatibility, and developable curvature.

Fabrication rationality therefore operates at the level of geometric generation itself rather than as a post-rationalization procedure applied afterward.

References

Müller, M. M.; Ben Amar, M.; Guven, J. "Conical Defects in Growing Sheets." *Physical Review Letters* 101, 156104, 2008.

Menges, A. "Performative Morphology in Architecture: Integrative Design Research by the Institute for Computational Design." *Serbian Architectural Journal*, vol. 5, no. 2, 2013, pp. 92–105.